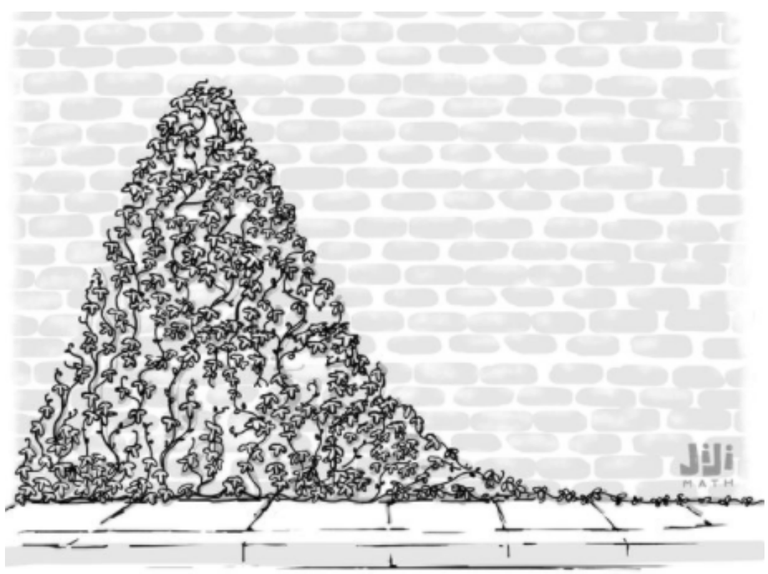
SPC

LESSON: Quality Methods - Special Discrete Distributions

Quality Methods
Special Discrete Distributions



Poisson Ivy

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Minitab

Special *Discrete* Random Variables

Binomial Distribution

- Series of n independent trials, two possible outcomes on each trial: success or failure, probability of success p on each trial is constant
- Sampling with replacement or large lot size
- X denotes the number of successes

Hypergeometric distribution

- Sampling from a finite population without replacement.
- X denotes the number of defective items
- Used in Acceptance Sampling

Poisson Distribution

- X denotes the number of random events that occur within a fixed unit of time, fixed unit of space, or fixed product unit

Binomial Random Variable

Let X represent the number of successes out of n independent trials, where the probability of success p is constant. Then the **probability mass function of X is**:

$$P(X=x)=p(x)=\binom{n}{x}\cdot p^{x}\cdot(1-p)^{n-x}$$

for $x = 0, 1, 2, \ldots, n$.

p = probability of "success" on any trial
n = number of trials
x = number of successes

Mean: $\mu = np$ **Variance:** $\sigma^2 = np(1-p)$

Binomial coefficient $\binom{n}{x}$ definition

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$

By Definition: $0! = 1$

The number of ways to choose x items out of n where order matters.

Example 1: $$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!(2!)} = 6$$

Example 2: $$\binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = \frac{6*5}{2} = 15$$

Sample Binomial RV Problem

Example 1. An assembly line produces a continuous product that is 3% defective. Let *X* represent the number of defectives out of 100 pieces randomly selected from the line.

(a) Write down the probability mass function of *X* with its support.

(b) What is the probability that there are exactly 2 defectives out of the 100 pieces selected? 0.225

(c) What is the probability that there is at least one defective? 0.9524

(d) How many defectives would you expect out of a sample of 100 pieces? Alternative E(X) formula

Poisson Random Variable

Let X be the number of defectives per a certain product unit, time period, or space. Then the probability mass function of X is:

$$p(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad x = 0, 1, 2, \ldots$$

where l is the mean or average number of events that happened over the specified product (defective rivets on a car door), volume (blemishes per sq yd of fabric), or time period (breakdowns per month).

Mean :$\mu = \lambda$ Variance : σ^2 =l

"Life is good for only two things: discovering mathematics and teaching mathematics." -- *Simeon Poisson*

Poisson Random Variable

Example 2. Suppose that typographical errors are made at the rate of 2 per page in *The Thorn*. Let X represent the number of typos on the first page. What is the probability that less than 3 typos appear on the first page? 0.677

Suppose this week's paper is 6 pages in total length. What is the probability that 4 or more typos appear in the 6 pages? Be careful – make sure the Poisson random variable is on the right "scale." 0.998

More examples ...

Example 3. Suppose that 24% of all customers who return their product registration form also enclose the customer satisfaction card with the box "very pleased" checked. To enhance customer satisfaction, sales personnel undergo a required training course. If 12 or more out of the next 25 product registrations are accompanied by satisfaction cards with the "very pleased" box checked, the training will be deemed successful. What is the probability the training will be viewed as successful?

Let X represent the number of "very pleased" boxes checked out of the next 25. 0.0076

Example 4. If a company that makes spandex fiber averages 8.8 knots per million meters of fiber, what is the probability that in a warp that uses a sample of 500,000 meters of fiber, there will be at most two knots?

Let X represent the number of knots in 500,000 meters of fiber. 0.185

Example 5. Identifying Discrete Distributions. Each of these random variables is best represented by one of the following distributions: Binomial or Poisson.

(a) Let X represent the number of out-of-control points that show up on an I-MR control chart out of 100 consecutive data points. The probability of obtaining an out-of-control point when the process is actually in statistical control is 0.0027. Assume plotted points are independent of each other.

(b) In a building, there are 10 stalls in the women's restroom. Suppose that the probability of a leaking toilet is 0.2. A "spot check" is performed on the 10 stalls before leaving for the evening. Let X represent the number of stalls that have a leaky toilet.

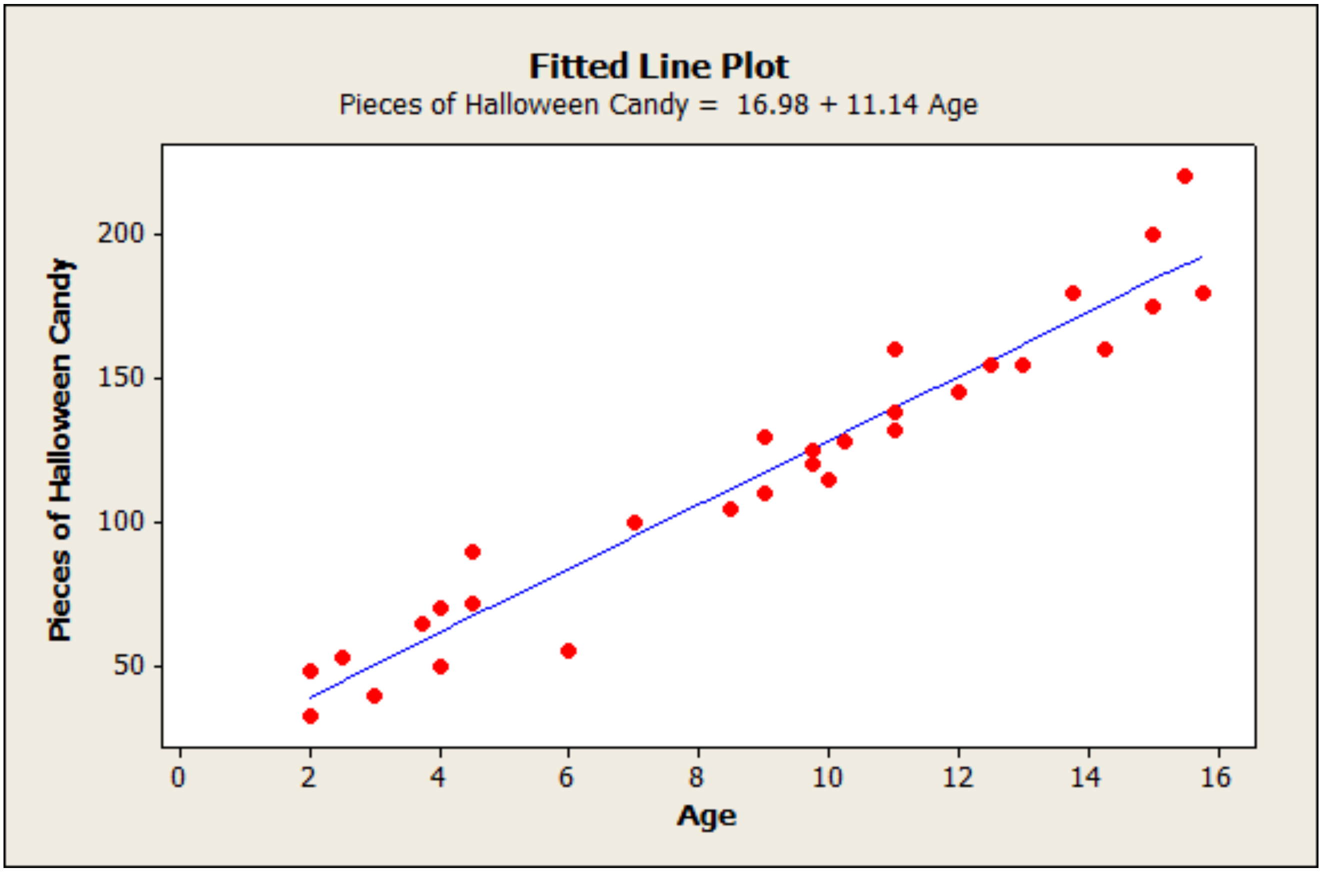
(c) Ever since Honey Boo Boo made "Go Go Juice" famous, Circle K convenient store can't keep enough Red Bull on their shelves to meet current demand. Let X represent the number of Red Bulls that Circle K sells in a given day, where the average is 30 per day.

(d) Let X represent the number of prescription errors at a pharmacy in a week, where the average number is 5 prescription errors per week.

(e) There are 175 faculty members at a university. The probability that a faculty member will get a flu shot is 0.6. Let X represent the number of faculty out of 175 who will get a flu shot this year. Faculty members make decisions to get flu shots independently of each other.

(f) When submitting a homework set online, 20% of students submit it without putting a name on it. I decide to take a random sample of 10 homework sets to check for students' names. Let X represent the number of sets out of the 10 that have don't have names on them.

(g) Let X represent the number of home foreclosures in a month in a particular city, where the average number is 15 homes per month.



(h) On average, a trick-or-treater gets 100 pieces of candy on a "good" Halloween evening (where good = nice temperature, no rain, active neighborhood). Let *X* represent the number of pieces of candy in a randomly selected trick-or-treaters bag.